Semi-Graphoid Properties of Variants of Epistemic Independence based on Regular Conditioning

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Goal:

to study the semi-graphoid properties of concepts of independence based on regular conditioning.

(With respect to credal sets that are general sets of Kolmogorovian-style probability measures.)
Semi-Graphoid Properties

Symmetry: \((X \perp \!\!\!\!\!\!\perp Y \mid Z) \Rightarrow (Y \perp \!\!\!\!\!\!\perp X \mid Z)\)

Redundancy: \((X \perp \!\!\!\!\!\!\perp Y \mid X)\)

Decomposition: \((X \perp \!\!\!\!\!\!\perp (W, Y) \mid Z) \Rightarrow (X \perp \!\!\!\!\!\!\perp Y \mid Z)\)

Weak union: \((X \perp \!\!\!\!\!\!\perp (W, Y) \mid Z) \Rightarrow (X \perp \!\!\!\!\!\!\perp Y \mid (W, Z))\)

Contraction: \((X \perp \!\!\!\!\!\!\perp Y \mid Z) \land (X \perp \!\!\!\!\!\!\perp W \mid (Y, Z)) \Rightarrow (X \perp \!\!\!\!\!\!\perp (W, Y) \mid Z)\)
Regular conditioning:

$$\mathbb{E}^> [f(X) | H] = \inf \{ \mathbb{E}_P [f(X) | H] : P \in \mathcal{K}(X) \text{ and } P(H) > 0 \}$$

whenever $\overline{P}(H) > 0$. 
Y is regular-epistemically irrelevant to $X$ given $Z$:

$$E^> [f(X) | y, z] = E^> [f(X) | z] \quad \text{whenever } \overline{P}(y, z) > 0.$$ 

Symmetry fails: “symmetrize” to get corresponding independence.
Theorem

If \((Y \text{ IR} X \mid Z)\) denotes regular-epistemic irrelevance of \(Y\) to \(X\) given \(Z\), then:

- \((X \text{ IR} Y \mid X)\) and \((Y \text{ IR} X \mid X)\);
- If \((X \text{ IR} W, Y \mid Z)\), then \((X \text{ IR} Y \mid Z)\);
- If \((X \text{ IR} W, Y \mid Z)\), then \((X \text{ IR} Y \mid W, Z)\) \([\text{NOTE: FAILS FOR de Finettian-conditioning!}]\);
- If \((Y \text{ IR} X \mid Z)\) and \((W \text{ IR} X \mid Y, Z)\), then \((W, Y \text{ IR} X \mid Z)\).
More results

Theorem

Regular-epistemic independence satisfies Symmetry and Redundancy.
Type-5 concepts

- Y is *type-5 epistemically irrelevant* to X given Z:

  \[ \mathbb{E}^> [f(X)|B, z] = \mathbb{E}^> [f(X)|z] \text{ whenever } \bar{P}(B, z) > 0. \]

Symmetry fails: “symmetrize” to get corresponding independence.
Even more results

Theorem
If \((Y \text{ IR } X \mid Z)\) denotes type-5 epistemic irrelevance of \(Y\) to \(X\) given \(Z\), then:

- \((X \text{ IR } Y \mid X)\) and \((Y \text{ IR } X \mid X)\);
- If \((X \text{ IR } W, Y \mid Z)\), then \((X \text{ IR } Y \mid Z)\);
- If \((X \text{ IR } W, Y \mid Z)\), then \((X \text{ IR } Y \mid W, Z)\);
- If \((W, Y \text{ IR } X \mid Z)\), then \((Y \text{ IR } X \mid Z)\);
- If \((W, Y \text{ IR } X \mid Z)\), then \((Y \text{ IR } X \mid W, Z)\);
And more results

Theorem

Type-5 independence and type-5 epistemic independence both satisfy Symmetry, Redundancy, Decomposition and Weak Union.
Also in the paper: complete and strong independence

- Complete independence satisfies all semi-graphoid properties.
- Strong independence satisfies Symmetry, Redundancy, Decomposition and Weak Union — but it fails Contraction!
Conclusion

- In this paper: a detailed map of semi-graphoid properties (all properties not mentioned fail...!).
- Confirmational/epistemic seem very weak... “type-5 condition” leads to better behavior.