The Joy of Probabilistic Answer Set Programming

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Goal:

to show that the credal semantics for Probabilistic Answer Set Programming (PASP) leads to a very useful modeling language.
Answer set programming (ASP)...

- A program is a set of rules such as

  \[\text{green}(X) \lor \text{green}(X) \lor \text{blue}(X) : \neg \text{node}(X), \text{not barred}(X).\]

- A fact is a rule with no subgoals:

  \[\text{node}(a)\.\]
Stable model semantics

- Herbrand base: all groundings generated by constants in the program.
- Minimal model is a model (interpretation that satisfies all rules) such that none of its subsets is a model.
- Answer set: a minimal model of the reduct (propositional program obtained by grounding, then removing rules with not, then removing negated subgoals).
A PASP program contains rules, facts, and *probabilistic facts*:

\[ 0.25 :: \text{edge}(\text{node1}, \text{node2}). \]

\[ 0.25 :: \text{edge}(\text{node2}, \text{node3}). \]

A *total choice* induces an Answer Set Program.
Acyclic propositional (Bayesian network)

0.01 :: trip.
0.5 :: smoking.
tuberculosis :- trip, a1.
tuberculosis :- not trip, a2.
0.05 :: a1. 0.01 :: a2.
cancer :- smoking, a3.
cancer :- not smoking, a4.
0.1 :: a3. 0.01 :: a4.
either :- tuberculosis.
either :- cancer.
test :- either, a5. 0.98 :: a5.
test :- either, a6. 0.05 :: a6.
Stratified programs

\begin{align*}
\text{edge}(X, Y) & : \neg \text{edge}(Y, X). \\
\text{path}(X, Y) & : \neg \text{edge}(X, Y). \\
\text{path}(X, Y) & : \neg \text{edge}(X, Z), \text{path}(Z, Y). \\
0.6 & : \text{edge}(1, 2). \\
0.1 & : \text{edge}(1, 3). \\
0.4 & : \text{edge}(2, 5). \\
0.3 & : \text{edge}(2, 6). \\
0.3 & : \text{edge}(3, 4). \\
0.8 & : \text{edge}(4, 5). \\
0.2 & : \text{edge}(5, 6). 
\end{align*}
PASP: Credal semantics

- A total choice may induce a program with many answer sets.

\[ \theta_1, \theta_2, \ldots \]

Probability of each total choice may be distributed freely over answer sets: semantics is a credal set that dominates an infinitely-monotone capacity.
PASP: Credal semantics

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- Probability of each total choice may be distributed freely over answer sets: semantics is a credal set that dominates an infinitely-monotone capacity.
Is there a three-coloring?
Three-coloring

\[
\begin{align*}
\text{red}(X) \lor \text{green}(X) \lor \text{blue}(X) & : \neg \text{node}(X). \\
\text{edge}(X, Y) & : \neg \text{edge}(Y, X). \\
\neg \text{colorable} & : \neg \text{edge}(X, Y), \text{red}(X), \text{red}(Y). \\
\neg \text{colorable} & : \neg \text{edge}(X, Y), \text{green}(X), \text{green}(Y). \\
\neg \text{colorable} & : \neg \text{edge}(X, Y), \text{blue}(X), \text{blue}(Y). \\
\text{red}(X) & : \neg \neg \text{colorable}, \text{node}(X), \textbf{not} \ \neg \text{red}(X). \\
\text{green}(X) & : \neg \neg \text{colorable}, \text{node}(X), \textbf{not} \ \neg \text{green}(X). \\
\text{blue}(X) & : \neg \neg \text{colorable}, \text{node}(X), \textbf{not} \ \neg \text{blue}(X). \\
\end{align*}
\]

Then: \( \mathbb{P}(\text{colorable}, \text{blue}(3)) = 0.976. \)
Interpretation

- Lower/upper probabilities: *sharp* probabilities with respect to appropriate questions.
- “What is the probability that I will be able to select a three-ordering where node 2 is red?”
  - Answer is $\overline{P}(\text{colorable, red}(2))$. 
In the paper:

Algorithm to compute lower/upper probabilities!
In short: PASP with credal semantics is a very powerful language.

- We can compute probabilities with some implicit quantification.