Natural Selection with Objective Imprecise Probability
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1. Natural selection sometimes produces patterns of behaviors in members of species $S_1$ that are imprecisely probabilistically distributed, conditional on perceived environmental conditions; precisely calibrated, probabilistic behaviors are too costly.

2. These behaviors form part of the environment for members of another species $S_2$ (predators, prey, competitors, disease vectors, etc.).

3. So the $S_2$ population’s environment includes imprecisely probabilistic conditions that can affect success in producing descendants.

4. $S_2$ is part of the environment of $S_1$, $S_3$, etc.

5. Thus natural selection often depends on objective imprecise probabilities.

Causal probability and erraticly
- **Causal probability**: Objective probability realized by a set of conditions, a chance setup (person tossing dice) producing outcomes, where manipulating some of these conditions (densities in the dice) manipulates probability and, usually, relative frequencies.
- I assume there are ways for causal probability to be realized by underlying deterministic dynamics, as is in dice tossing.
- **Erratic setups** have outcomes but don’t realize probability of any kind, at the level of the setup. (What’s the objective probability that the percentage of ink in pieces of paper in pockets of the next ten people who attend a talk at ISIPTA lies within such and such bounds?)
- Natural selection depends on probabilities of survival and reproduction for organisms with different traits in an environment. If environments varied erratically, these probabilities could be imprecise.

Behavioral imprecision (premise 1)
- Let environmental states have precise objective probabilities.
- Natural selection should favor traits producing optimal behaviors conditional on perceptions of environmental state.
- Behavior narrowly distributed around an optimum is expensive: Nervous systems, muscles, bone, etc. require time to build, and energy to maintain.
- Good, imperfect: Probabilistic behavior, miscalibrated mean.
- Good enough, less perfect: Imprecisely probabilistic behavior.
- Note: If our behavior doesn’t result from precise credences and utilities, why should organisms be better?

Imprecise fitness and decision rules
- Trait d: dig deep burrows, fitter in dry periods
  Trait s: dig shallow burrows: fitter in wet periods
- Fitness $w(x)$ for $x = d, s$ in environments $e$: $w(x) = E_x w_e(x)$.
- If environments vary erratically: lower/upper (objective) previsions, supremum $w(x)$, precise fitnesses.
- Trait $A_1$ is fitter than trait $A_2$ if $A_1$ interval dominates $A_2$: $A_1 \sqsubset A_2$ iif $w(A_1) > w(A_2)$.
- Trait $A_1$ is fitter than trait $A_2$ if environments vary erratically so that the entire population experiences the same environment at $t$, and $A_1$ dominates across population-wide environments: Then $A_1$ is fitter that $A_2$ iif $(\forall e) w_e(A_1) > w_e(A_2)$.
- Other decision rules don’t seem relevant. E.g. E-admissible traits won’t necessarily be selected for. These are traits such that there is some particular environment that makes all of them at least as fit as all other traits: \{ $A_i : (\exists e) (\forall A_j) E_x(A_i) \geq E_x(A_j)$ \}.

Im/precise Wright-Fisher models
**Precise**: Simple model of change in allele frequencies. (Precise) probability of transition from $i$ to $j$ alleles: $p_{ij} = \left(\frac{1}{2^n}\right)^2 (1 - \eta)^{2N-1}$, where the (precise) probability of an $A$ allele being chosen is: $\eta = \frac{w_{AB} + w_{BA} (2N-1)}{w_{AB} + w_{BA} (2N-1) + w_{BB} (2N-1)}$.

Here $A$ is fitter than $B$, so probable frequencies of $A$ increase, $w_{AA} = 1.0$, $w_{AB} = 0.9$, $w_{BB} = 0.7$: